Journal of Sound and Vibration (2002) **254**(1), 175–178
doi:10.1006/jsvi.2001.4076, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF A CIRCULAR PLATE OF RECTANGULAR ORTHOTROPY WITH A SECANT SUPPORT

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(*Received* 20 *August* 2001)

1. INTRODUCTION

The present study deals with the solution of the title problem in the case of clamped and simply supported edges, see Figure 1. For the sake of generality it is assumed that the plate carries a centrally located concentrated mass. This note deals with an extension of the problem tackled in reference [1], where an isotropic plate was considered.

It may be of interest to recall that continuous, rectangular plates executing small amplitude, transverse vibrations have been treated in many excellent papers [2, 3]. On the other hand very limited information is available in the open literature on vibrating continuous plates of other shapes, e.g., a circular plate with a secant support.

Two independent solutions are obtained using the optimized Rayleigh–Ritz method and the finite element approach [4].

2. APPLICATION OF THE OPTIMIZED RAYLEIGH–RITZ METHOD

When the structural system under study vibrates in one of its normal modes the displacement amplitude satisfies the well-known functional:

$$J(W) = \iint_{\bar{P}} (D_1 W_{\bar{x}^2}^2 + 2D_1 \upsilon_2 W_{\bar{x}^2} W_{\bar{y}^2} + D_2 W_{\bar{y}^2}^2 + 4D_k W_{\bar{x}\bar{y}}^2)\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - \rho h \omega^2 \iint_{\bar{P}} W^2\,\mathrm{d}\bar{x}\,\mathrm{d}\bar{y} - M\omega^2 W^2(0,0) \quad (1)$$

and appropriate boundary conditions.

Substituting the dimensionless variables $x = \bar{x}/a$ and $y = \bar{y}/b$ into equation (1), one obtains after some straightforward manipulations:

$$\frac{a^2}{D_1} J(W) = \iint_P (W_{x^2}^2 + 2\upsilon_2 W_{x^2} W_{y^2} + D_2' W_{y^2} + 4D_k' W_{xy}^2)\,\mathrm{d}x\,\mathrm{d}y - \Omega^2 \left[\iint_P W^2\,\mathrm{d}x\,\mathrm{d}y + \pi\mu W^2(0,0) \right], \quad (2)$$

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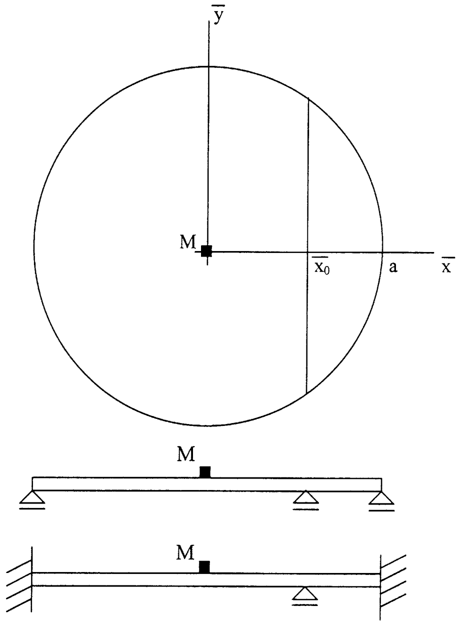

Figure 1. Vibration orthotropic circular plate with a secant support.

where $D'_2 = D_2/D_1$, $D'_k = D_k/D_1$, $\Omega^2 = \rho h a^4/D_1\,\omega^2$, $\mu = M/M_p = M/\rho h A_p$, $A_p = \pi a^2$. Following reference [1] one assumes

$$W \cong W_a = [C_1(\alpha_1 r^p + \beta_1 r^2 + 1) + C_2(\alpha_2 r^{p+2} + \beta_2 r^4 + r^2)](x - x_0) = \sum_{j=1}^{2} C_j \varphi_j(x, y). \quad (3)$$

The α_i's and β_i's are determined following reference [1] in order to satisfy the essential boundary conditions. Applying Ritz minimization condition one obtains

$$\frac{a^2}{2D_1}\frac{\partial J}{\partial C_i} = \sum_{j=1}^{2}\left\{\iint_P \left[\varphi_{jx^2}\varphi_{ix^2} + \upsilon_2(\varphi_{jy^2}\varphi_{ix^2} + \varphi_{jx^2}\varphi_{iy^2}) + D'_2\varphi_{jy^2}\varphi_{iy^2} + 4D'_k\varphi_{jxy}\varphi_{ixy}\right]\mathrm{d}x\,\mathrm{d}y\right.$$
$$\left. - \Omega^2\left[\iint_P \varphi_j\varphi_i\,\mathrm{d}x\,\mathrm{d}y + \pi\mu\varphi_j(0)\varphi_i(0)\right]\right\}C_j = 0, \quad (4)$$

which leads, finally, to a determinantal equation whose lowest root is the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D_1}\,\omega_1 a^2$.
Minimizing Ω_1 with respect to the exponential parameter "p" one obtains an optimized value of Ω_1.

3. NUMERICAL RESULTS

Fundamental frequency coefficients were determined for $D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$ and $\upsilon_2 = 0{\cdot}3$. For the isotropic case the Poisson ratio was taken to be equal to 0·3. When using the finite element procedure half of the plate domain was subdivided in 10 122 elements (see Figure 2).

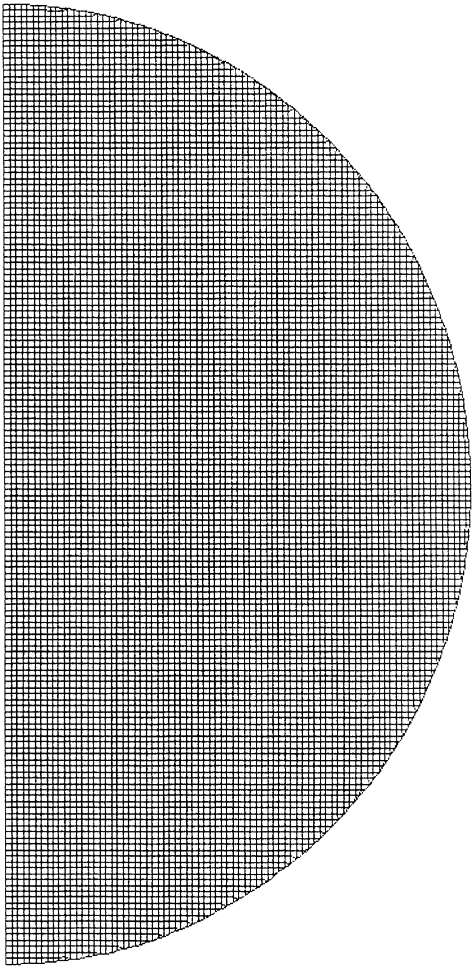

Figure 2. Finite element mesh corresponding to half of the plate.

Table 1 depicts the values of Ω_1 for simply supported and clamped, isotropic and orthotropic plates as a function of x_0/a. There is good agreement between finite element and analytical results, specially for $x_0/a \leqslant 0{\cdot}6$. Table 2 shows the fundamental eigenvalue in the case where a concentrated mass is rigidly attached to the plate center. Only the analytical approach has been used for this situation.

It is important to point out that when the secant support coincides with the diameter of the plate the values obtained for the isotropic situation by means of the finite element agree admirably well with the exact eigenvalues corresponding to the first antisymmetric mode of the plate [5].

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur.

TABLE 1

Fundamental frequency coefficients of simply supported and clamped plates: comparison of finite element and analytical results ($M/M_p = 0$)

	Finite element values				Anlytical results			
	Simply supported		Clamped		Simply supported		Clamped	
x_0/a	Isotropic	Orthotropic	Isotropic	Orthotropic	Isotropic	Orthotropic	Isotropic	Orthotropic
0	13·910	13·507	21·262	20·282	14·064	13·685	21·272	20·505
0·1	13·121	12·731	19·934	18·986	—	—	—	—
0·2	11·753	11·382	17·814	16·901	12·261	11·873	18·200	17·409
0·3	10·489	10·131	15·983	15·040	—		—	—
0·4	9·423	9·072	14·400	13·501	9·765	9·360	14·697	13·840
0·5	8·539	8·189	13·160	12·246	—	—	—	
0·6	7·804	7·448	12·172	11·232	8·174	7·760	12·863	11·948
0·7	7·185	6·817	11·397	10·421	—	—	—	
0·8	6·651	6·263	10·811	9·791	7·209	6·788	11·951	10·960
0·9	6·156	5·737	10·407	9·339	—	—	—	—
1	4·942	4·438	10·216	9·091	—	—	—	—

TABLE 2

Fundamental frequency coefficients of simply supported and clamped plates carrying a centrally located concentrated mass: analytical results

	$M/M_p = 0{\cdot}2$				$M/M_p = 0{\cdot}4$			
	Simply supported		Clamped		Simply supported		Clamped	
x_0/a	Isotropic	Orthotropic	Isotropic	Orthotropic	Isotropic	Orthotropic	Isotropic	Orthotropic
0	14·064	13·685	21·272	20·505	14·064	13·685	21·272	20·505
0·2	9·846	9·506	13·287	12·700	8·401	8·100	10·893	10·409
0·4	6·688	6·376	8·963	8·425	5·376	5·118	6·992	6·570
0·6	5·329	5·030	7·431	6·888	4·228	3·984	5·734	5·314
0·8	4·630	4·336	6·748	6·196	3·660	3·423	5·189	4·763

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